GCE

Mathematics B MEI

H640/03: Pure Mathematics and Comprehension

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.
BP	Blank Page
Seen	
Highlighting	

6. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1			$\frac{6x}{5} > 6$ or $x > 30 - 5x$	M1	1.1	An initial step to getting all terms in x on one side. Allow incorrect inequality signs
			$x > 5$	A1	1.1	Correct completion
				[2]		

Question			Answer	Marks	AO	Guidance
2	(a)		$y = \sqrt{1+2x} \Rightarrow y^2 = 1 + 2x$	M1	1.1	Starting to work through inverse processes Allow equivalent processes e.g. swapping x and y at other stages
			$\left[f^{-1}(x) =\right] \frac{x^2 - 1}{2}$ oe	A1	1.1	
			The domain is $x \geq 0$	B1	2.2a	**B0** if $y \geq 0$
				[3]		
2	(b)		It is many-to-one **or** Is not one-to-one **or** It is one-to-many	E1	2.4	Could be via an example e.g. $x = 2$ and $x = -2$ give the same value of g(x) Give BOD for use of 'it' provided it does not contradict e.g. g(x) is one-to-many does not score isw after a correct answer
				[1]		

Question	Answer	Marks	AO	Guidance	
3	**DR** Area below curve = $\int_0^1 x^5\,\mathrm{d}x$	**M1**	**1.1**	Any correct expression for area below curve. Condone missing dx but limits required or used somewhere	The two steps could be done in either order Words not required. Reverse limits scores M0A0
	Area below curve = $\left[\frac{x^6}{6}\right]_0^1 = \frac{1}{6}$	**A1**	**1.1**		
	Area to left of curve = $1 - \frac{1}{6}$	**M1**	**3.1a**	1 minus *their* area below curve	Or Area to left of curve = $\int_0^1 y^{\frac{1}{5}}\,\mathrm{d}y = \frac{5}{6}\left[y^{\frac{6}{5}}\right]_0^1 = \frac{5}{6}$
	$\left[\frac{\text{Area to the left of the curve}}{\text{Area below curve}} =\right] \frac{\frac{5}{6}}{\frac{1}{6}} = 5$	**E1**	**2.1**	Convincing completion (answer given)	Dep. on previous 3 marks
		[4]			

Question	Answer	Marks	AO	Guidance
4	**DR** e.g. $\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}$	**M1**	**3.1a**	Rationalising denominators At least one example of multiplying by e.g. $\frac{\sqrt{2}-1}{\sqrt{2}-1}$
	$\frac{\sqrt{2}-1}{2-1}+\frac{\sqrt{3}-\sqrt{2}}{3-2}+\frac{2-\sqrt{3}}{4-3}$ $2-1=1$	**A1**	**1.1**	Convincing completion including all steps including the denominators of 1 or better
	Alternative method			
	$\frac{(\sqrt{3}+\sqrt{2})(2+\sqrt{3})+(\sqrt{2}+1)(2+\sqrt{3})+(\sqrt{2}+1)(\sqrt{3}+\sqrt{2})}{(\sqrt{2}+1)(\sqrt{3}+\sqrt{2})(2+\sqrt{3})}$ $\frac{\sqrt{3}+3+2\sqrt{2}+\sqrt{6}+2\sqrt{2}+\sqrt{6}+2+\sqrt{3}+\sqrt{6}+2+\sqrt{3}+\sqrt{2}}{(\sqrt{6}+2+\sqrt{3}+\sqrt{2})(2+\sqrt{3})}$ $\frac{3\sqrt{6}+4\sqrt{3}+5\sqrt{2}+7}{2\sqrt{6}+\sqrt{18}+4+2\sqrt{3}+2\sqrt{3}+3+2\sqrt{2}+\sqrt{6}}$			
	$\frac{3\sqrt{6}+4\sqrt{3}+5\sqrt{2}+7}{3\sqrt{6}+4\sqrt{3}+3\sqrt{2}+2\sqrt{2}+7}$	**M1**		Putting over a common denominator and simplifying all like terms
	$\frac{3\sqrt{6}+4\sqrt{3}+5\sqrt{2}+7}{3\sqrt{6}+4\sqrt{3}+5\sqrt{2}+7}=1$	**A1**		Convincing completion including all steps
		[2]		

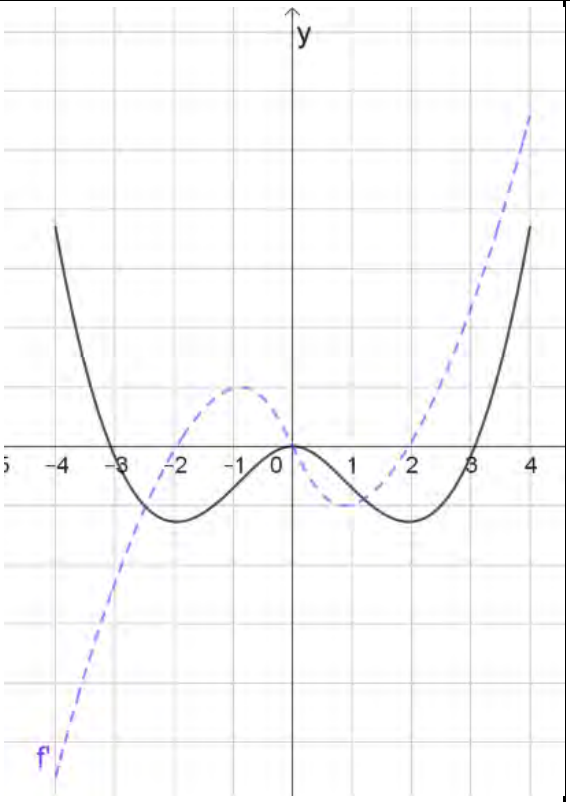
Question			Answer	Marks	AO	Guidance
5			**DR** $u = x + 1$			
			$\mathrm{d}u = \mathrm{d}x$	**M1**	**1.1**	oe soi here or at any point before first A mark
			$\int_{x=c}^{x=c+4} \left(\frac{1}{u} - \frac{1}{u^2}\right) \mathrm{d}u$	**M1**	**1.1**	This mark is for using the substitution, ignore limits and use of du for this mark. So may see no du but do not award if dx. Condone $\int \frac{u-1}{u^2} \mathrm{d}u$.
			$\left[ln u + \frac{1}{u}\right]_{x=c}^{x=c+4}$	**M1**	**1.1**	This mark is for attempt to integrate their expression and must involve ln u or a power of u Ignore limits for this mark.
			$\ln(c+5) + \frac{1}{c+5} - \ln(c+1) - \frac{1}{c+1}$	**A1**	**2.1**	oe. Correct integral in terms of c Condone recovery of missing brackets
			Attempting to solve either *their* $\frac{c+5}{c+1} = 3$ **or** *their* $\frac{1}{c+5} - \frac{1}{c+1} = -\frac{1}{3}$ oe	**M1**	**3.1a**	
			$c = 1$ and checking their solution in other equation Or solve both and get same answer	**A1**	**1.1**	www If they ignore the instruction and use a different integration method to perform substitution they can have a SC1 if they get to the correct answer. Also the final M1A1 are still available
				[6]		

Question			Answer	Marks	AO	Guidance
6			**DR**			
			$\tan^2 x - 3 = 2\tan x$	**M1^**	**3.1a**	Multiplying through by $\tan x$
			$\tan^2 x - 2\tan x - 3 = 0$	**M1^**	**1.1**	Get all 3 terms of quadratic on one side and zero on the other
			$(\tan x - 3)(\tan x + 1) = 0$ $\tan x = 3\ ,\ \tan x = -1$	**M1***	**1.1**	Solve 3 term quadratic to get both values of $\tan x$ **Must** see either use of the formula, factorisation or completing the square. Allow 1 error in method Condone missing '= 0'
			71.6°, 251.6°, 135°, 315°	**A1^** **A1***	**1.1** **1.1**	**A1^** for any two roots correct (dep. on **M2^**) **A1*** for all roots with no additional solutions (dep. on **M1***) Accept 72 and 252
			Alternative solution			
			$\frac{\sin x}{\cos x} - 3\frac{\cos x}{\sin x} = 2$	**M1^**	**3.1a**	Use of $\tan x = \frac{\sin x}{\cos x}$
			$\sin^2 x - 2\sin x\cos x - 3\cos^2 x = 0$	**M1^**	**1.1**	Get all 3 terms on one side and zero on the other
			$(\sin x - 3\cos x)(\sin x + \cos x) = 0$ $\tan x = 3\ ,\ \tan x = -1$	**M1***	**1.1**	Factorise to get both values of $\tan x$ **Must** see factorisation. Allow 1 error in method Condone missing =0
			71.6°, 251.6°, 135°, 315°	**A1^** **A1***	**1.1** **1.1**	**A1^** for any two roots correct (dep. on **M2^**) **A1*** for all roots with no additional solutions (dep. on **M1***) Accept 72 and 252
				[5]		

	Question		Answer	Marks	AO	Guidance
7			$2\sin 4\theta\cos 4\theta\tan 4\theta+\cos 8\theta$	**M1**	**3.1a**	e.g. Use of $\sin 2A=2\sin A\cos A$ Use of means more than just quoting the formula - usually it will be part of an argument Small angle approximations score M0
			$2\sin 4\theta\cos 4\theta\dfrac{\sin 4\theta}{\cos 4\theta}+\cos 8\theta$	**M1**	**3.1a**	e.g. Use of $\tan A=\dfrac{\sin A}{\cos A}$ The 2 double angle formulas along with the tan identity may be applied in any order. If proof is sound award the 3 marks for equivalent steps.
			$2\sin^2 4\theta+1-2\sin^2 4\theta=1$	**A1**	**2.1**	e.g. Use of any correct identity for $\cos 2A$ and convincing completion to given result Working it through as an equation with the ' = 1 ' is not 'convincing' **Alternative incorrect approach**. If sin 8θ = 4sin2θ, and sin 2θ is then expanded then **SC1** can be awarded. A similar approach applies to cos 8θ. However the **SC1** can only be awarded once.
				[3]		

Question			Answer	Marks	AO	Guidance
8	(a)		**DR** $\left[\cos x+\sqrt{3}\sin x = R\sin\left(x+\alpha\right)\right]$ $\Rightarrow R\cos\alpha=\sqrt{3}\ ,\ R\sin\alpha=1$	**M1**	**1.1a**	M0 if R missing but next M mark available. Must be using α and not x.
			$\tan\alpha=\dfrac{1}{\sqrt{3}}$	**M1**	**1.1** **1.2**	Allow other equivalent methods using *their* values of sin and cos e.g. drawing a right angled triangle or simply dividing the two equations. Allow equivalent surds
			$\left[\alpha\right]=\dfrac{\pi}{6}$	**A1**		Dependent on M2
			$\Rightarrow R^2=4, R=2$	**B1**	**1.1**	soi anywhere
				[4]		
8	(b)		**DR** $\cos x+\sqrt{3}\sin x=\sqrt{3}\Rightarrow 2\sin\left(x+\dfrac{\pi}{6}\right)=\sqrt{3}$	**M1**	**3.1a**	Use of *their* result from **(a)**
			$\sin\left(x+\dfrac{\pi}{6}\right)=\dfrac{\sqrt{3}}{2}$	**M1**	**1.1**	Value for a trig function following on from first step
			$x+\dfrac{\pi}{6}=\dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$	**M1**	**1.1**	At least one value for *their* $x+\dfrac{\pi}{6}$. Condone use of inequality symbols if recovered. Condone working consistently in degrees up to this point.
			$x=\dfrac{\pi}{6}\ ,\ \dfrac{\pi}{2}$	**A1**	**1.1**	Both roots correct in radians only. If extra roots then A0 Candidates who square both sides and solve from there, should check their solutions and discard any erroneous ones. This gains a maximum of M3A0. If no check then M2A0 max.
				[4]		

Question			Answer	Marks	AO	Guidance
9	(a)		$f(1) = -1$ $f(2) = 13.75$ or $\frac{55}{4}$ so there is a change of sign	B1 B1	1.1 1.1	Finding f(1) or f(2) Completion to show change of sign with explanation.
				[2]		
9	(b)		Value of x in the range $1.125 < x < 1.25$	B1	2.2a	Any value in this range. Candidates may give the range.
				[1]		
9	(c)		$f(1.15) = -0.09$ so $x \approx 1.2$ (cao)	B1	2.2a	Justified by their calculations (which may not necessarily use 1.15).
				[1]		
9	(d)	(i)	There is a change of sign;	E1	2.4	Incorrect maths (e.g. it implies a y-intercept) **B0**
				[1]		
9	(d)	(ii)	Clear and correct explanation	E1	2.4	E.g. • The function is undefined for $x = \frac{2}{3}$ [and it looks as if the spreadsheet is homing in on this value] • Accept 'discontinuous' or 'asymptote' for 'undefined' • Fig. 9.1 shows that there is only one root. • Could refer to the table (e.g. f(x) values diverging) isw after a correct answer
				[1]		

Question			Answer	Marks	AO	Guidance
10				B1 B1 B1	3.1a 2.2a 2.2a	Zeroes at 0 and close to -2 and 2 (and no others) Rotational symmetry about the origin (mark intent) General shape (actual values of gradient may be wrong)
				[3]		

Question			Answer	Marks	AO	Guidance
11	(a)		$\frac{dy}{dx} = x\cos x$ oe	B1	1.1	
			$x\cos x = 1$	M1	1.1	For *their* $\frac{dy}{dx} = 1$
			$\frac{1}{x} = \cos x$ so $\frac{1}{x} - \cos x = 0$	A1	2.1	Convincing completion to given answer
				[3]		

11	**(b)**		For $f(x)=\frac{1}{x}-\cos x$, $f'(x)=-\frac{1}{x^2}+\sin x$	**M1**	**3.1a**	Differentiation (for this mark look for a power of x and a term in sin x or cos x with at least one term correct)
			Iteration $x_{n+1}=x_n-\frac{\left(\frac{1}{x_n}-\cos x_n\right)}{\left(-\frac{1}{x_n^2}+\sin x_n\right)}$	**A1**	**1.1**	oe, e.g. $x_{n+1}=x_n-\frac{\left(x_n-x_n^2\cos x_n\right)}{\left(-1+x_n^2\sin x_n\right)}$ **(The subscripts are needed)**
			Suitable starting value	**M1**	**3.1a**	Starting values from 3.6 to 6.1 inclusive work for the given iteration but there may be other values which also give the required root and should be awarded.
			4.9172	**A1**	**1.1**	BC – candidates need not show intermediate iterations If a value outside the expected starting value is used and it converges to 4.9172 then **M1A1** is awarded (no need to check) awrt 4.9172
			Alternative method			
			For $f(x)=x\cos x-1$, $f'(x)=\cos x-x\sin x$			
				M1		Differentiation
			Iteration $x_{n+1}=x_n-\frac{\left(x_n\cos x_n-1\right)}{\left(\cos x_n-x_n\sin x_n\right)}$	**A1**		oe, e.g. $x_{n+1}=\frac{\left(1-x_n^2\sin x_n\right)}{\left(\cos x_n-x_n\sin x_n\right)}$
			Suitable starting value	**M1**		Starting values from 3.6 to 6.1 inclusive work for the given iteration but there may be other values which also give the required root.
			4.9172	**A1**		BC – candidates need not show intermediate iterations If a value outside the expected starting value is used and it converges to 4.9172 then **M1A1** is awarded awrt 4.9172
				[4]		

11	**(c)**		The gradient is close to zero so the next iteration is a long way from the root **or** As it is a turning point, the starting value is invalid as you cannot divide by 0 **or** The iteration converges to a different root.	**B1**	**3.2b**	Explanation referring to gradient of curve or to convergence to a different root. Not just that it is close to another root isw after a correct answer
				[1]		

Question			Answer	Marks	AO	Guidance
12	(a)		**DR** $\frac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	**M1**	**3.1a**	For differentiation of y wrt. θ, may be as part of $\frac{dy}{dx}$
			$\sin\theta + 2\sin\theta\cos\theta = 0$	**M1**	**3.1a**	For $\frac{dy}{d\theta} = 0$ or $\frac{dy}{dx} = 0$ and use of sine or cosine double angle formula. If $\frac{dx}{d\theta}$ is used incorrectly then give **M0**
			$\mathbf{\sin\theta(1+2\cos\theta) = 0}$	***M1**	**1.1**	All on one side and factorised. If they have divided through by $\sin\theta$ then $\mathbf{1 = -2\cos\theta}$ or $\mathbf{1+2\cos\theta = 0}$ will earn this mark
			$\sin\theta = 0$ or $\cos\theta = -\frac{1}{2}$	**DM1**	**1.1**	Dependent on the previous method mark At least one trig value Condone if divides through by sin.
			$\theta = 0$, $\frac{2\pi}{3}$, π , $\frac{4\pi}{3}$	**A1** **A1**	**1.1** **1.1**	**www** **A** marks dependent on **M4** At least 2 correct values All four values correct with no extras
			(2, 3)			
			(2, -1)	**A1**	**3.2a**	**www** At least 2 correct coordinates
			$\left(2-\frac{\sqrt{3}}{2},\ -1.5\right)$			oe, must be exact form
			$\left(2+\frac{\sqrt{3}}{2},\ -1.5\right)$	**A1**	**3.2a**	**www** All four coordinates correct with no extras oe, must be exact form
				[8]		

Question			Answer	Marks	AO	Guidance
12	(b)		$x = 2$	B1	2.2a	
				[1]		

Question			Answer	Marks	AO	Guidance
13			$t_1 t_2 = 1 \times -3 = -3$	B1	2.5	Convincingly showing that the formula gives −3
				[1]		

Question			Answer	Marks	AO	Guidance
14			$t_1^2 + t_2^2 + t_1 t_2 + \frac{1}{2} = 1^2 + (-3)^2 - 3 + \frac{1}{2}$ $= 1 + 9 - 3 + \frac{1}{2}$ $= 7.5$	B1	2.5	Convincingly showing that the formula gives 7.5
				[1]		

Question			Answer	Marks	AO	Guidance
15	(a)		$\frac{dy}{dx} = 2ax + b$	M1	1.1	Allow $2at + b$
			$y - (at^2 + bt + c) = (2at + b)(x - t)$ **or** $at^2 + bt + c = (2at + b)t + k$	M1	1.1	Use of a form of equation of straight line with *their* gradient and $x = t$
			$y = (2at + b)x - 2at^2 - bt + at^2 + bt + c$ So $\boldsymbol{y = (2at + b)x - at^2 + c}$	A1	2.1	Convincing completion to correct result
			Alternative method			
			The line and curve cross where $\boldsymbol{ax^2 + bx + c = (2at + b)x - at^2 + c}$	M1		
			$\boldsymbol{ax^2 - 2atx + at^2 = 0}$	M1		Getting everything to one side
			$a(x - t)^2 = 0$ Line touches curve when $x = t$ so it is the required tangent	A1		Convincing completion to correct result
				[3]		
15	(b)		Tangents cross where $\boldsymbol{(2ax_P + b)x - ax_P^2 + c = (2ax_Q + b)x - ax_Q^2 + c}$	M1	2.1	Use of tangent formula with distinct values of t Condone alternative notation for co-ordinates. e.g. P and Q, m and n, and other subscript versions
			$\boldsymbol{(2ax_P - 2ax_Q)x = ax_P^2 - ax_Q^2}$	M1	1.1	Getting terms in x on one side
			$x = \frac{x_P^2 - x_Q^2}{2(x_P - x_Q)}$ $= \frac{(x_P + x_Q)(x_P - x_Q)}{2(x_P - x_Q)} = \frac{x_P + x_Q}{2}$	A1	2.1	Convincing completion to given result. Accept the final result in terms of *their* variables.
				[3]		

Question	Answer	Marks	AO	Guidance
16	$a\left(\frac{x_P+x_Q}{2}\right)^2+b\left(\frac{x_P+x_Q}{2}\right)+c-a\left(\frac{x_P-x_Q}{2}\right)^2$ $=a\left\{\left(\frac{x_P+x_Q}{2}\right)^2-\left(\frac{x_P-x_Q}{2}\right)^2\right\}+b\left(\frac{x_P+x_Q}{2}\right)+c$ $=a\left(\frac{x_P+x_Q}{2}+\frac{x_P-x_Q}{2}\right)\left(\frac{x_P+x_Q}{2}-\frac{x_P-x_Q}{2}\right)+b\left(\frac{x_P+x_Q}{2}\right)+c$ $=ax_Px_Q+b\left(\frac{x_P+x_Q}{2}\right)+c$	**M1** **A1**	**1.1** **2.1**	Dealing with the a terms convincingly by e.g. collecting terms with factor a, expanding the brackets and collecting etc. Squaring brackets or difference of two squares demonstrated to give a convincing completion to given result
		[2]		

Question			Answer	Marks	AO	Guidance
17			$\frac{dy}{dx} = 2x$	M1	1.1	Accept $2t$
			Gradient of normal is $-\frac{1}{2t}$	M1	1.1	For use of negative reciprocal oe
			$y - t^2 = -\frac{1}{2t}(x - t)$ So $y = -\frac{x}{2t} + t^2 + \frac{1}{2}$	A1	2.1	For convincingly reaching given result Any error seen is **A0**.
			Alternative method for final mark			
			For the given line when $x = t$, $y = -\frac{1}{2} + t^2 + \frac{1}{2} = t^2$ and the gradient is $-\frac{1}{2t}$	A1		
				[3]		

Question			Answer	Marks	AO	Guidance
18			$y = 6x^2 - 7x + 1 - \frac{6 \times 5^2}{4}$	M1	3.1a	Use of result from line 19 of article
			$y = 6x^2 - 7x - 36.5$	A1	1.1	oe $\frac{73}{2}$ Accept a fully correct solution by an alternative method for **M1A1**. isw after a correct answer
				[2]		

Additional Guidance

This document provides additional information for some questions and should be used alongside the final MS. Please refer also to MS notes for examiners, which is the official CI introduction to the mark scheme. Please note in particular:

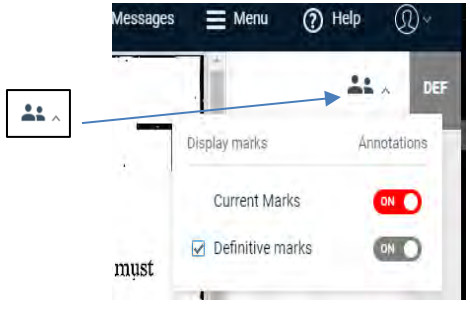
- Please use M, A and B, etc, as appropriate for the Standardisation scripts; ticks and crosses will often be sufficient later but responses that are not awarded either 0 or full marks should be annotated with enough M, B and A marks to show where marks have been earned. Even fully correct responses must have some annotation to show they have been seen.
- Where a candidate has crossed out a complete part of a question, it should be marked provided that it has not been replaced. If a candidate attempts more than one solution, you should mark the *most complete* solution, or if equal in this respect, you should mark what appears to be the *last* of these.
- Where a candidate has misread a **number** or **sign** in part of a question, and used that value consistently throughout that part, provided that number does not alter the difficulty, award all marks earned and deduct just 1 A or B mark for the misread. No misread is allowed if a candidate misreads a word or misunderstands a question or misreads their own writing.
- Any correct method should be accepted, unless disallowed by the wording of a question.

The rubric at the start of questions 3, 4, 5, 6, 8 and 12a states '**In this question you must show detailed reasoning.**' Candidates must show the steps required by the MS to gain the marks. If a step is missing then the final mark cannot be awarded.

There is one additional page attached before Q1. Please check this for working before starting to mark a script and link any working there to the relevant question. If they are blank, please annotate with either BP or SEEN.
The answer space for question 12(a) covers all of page 10 and all of page 11. Please check page 11; if anything is written there mark it, if not, please annotate with BP or SEEN.
Some pages may have working at the top of the page and then a large space underneath - please check to the bottom of each page. Additionally, sometimes candidates will work down to the bottom of the space and then finish their working back up at the top which may have disappeared of the top of your screen, so please check.
Generally, candidates need to collect like terms for final accuracy marks but equivalent (unsimplified) fractions are fine. Some questions have specific forms for the final answer so always check what is acceptable in each case.

Practice scripts have comments in the dialogue box at the bottom of each question as appropriate - please read them even if you agree with the marks as they can provide additional insight. To access the definitive marks on Practice scripts, you toggle the icon at the top of the marking pane. You must never leave a comment in the dialogue box.

When […] appears in the mark scheme, it means … is not required for the mark, it's there for information only.



Question specific notes

- *Some answers in the MS gain E1. In all cases please use B1 for these marks if awarded and B0 if not awarded.*
- *In several questions the final answer is given in the question, so the final mark is awarded for a 'convincing completion'. You must see correct working leading the given answer.*
- *When awarding M1 usually allow the mark if there is one error in the working, unless the MS specifies something different for that question .*

Section A

Q1. You need to annotate page 16 with either BP or SEEN if it is blank or link any working to the correct question part.
This is a straightforward inequality question. The M1 is given for seeing either of the two inequalities in the MS. Allow the M1 even if they use the incorrect signs. The A1 is for $x > 5$ only.

Q2a. Candidates may swap x and y at any stage, so you can allow the M1 for $x = \sqrt{(1+2y)}$ or $x^2 = 1 + 2y$.
The A1 is given for the correct answer or equivalent, so $\frac{x^2}{2} - \frac{1}{2}$ is alright. They do not need to write $f^{-1}(x)$ as it is given in the answer space (even if they do not put their answer there).
The B1 is given for giving the correct domain and may not be in the answer space allocated to it so allow it anywhere. But do not allow $y \geq 0$, $f^{-1}(x) \geq 0$ or $x > 0$

Q2b. We are allowing any of the three forms for the answer if using the word 'It'. 'It' could refer to the function g(x) or it's inverse $g^{-1}(x)$, so give B1 BOD if you see 'It is one-to-many'. But if the candidate specifically says 'g(x) is one-to-many' then that is incorrect and scores B0.

Q3. This is the first Detailed Reasoning (DR) question the paper.
The first M1 is for giving any correct expression for the area below the curve OR the area to the left of the curve (as they could do it in either order). We will condone missing dx but we need to see the correct limits $\int_0^1 \square$ either here or used in the calculation. Reversed limits will therefore score M0 A0 (can never give M0 A1).
The A1 is for showing the correct expression and the answer 1/6.
The second M1 is for finding the other area as 1 – their first area (even if their answer is negative).
The final B1 is dependent on the other 3 marks and is for forming the fraction with both 5/6 and 1/6 and getting the answer 5. The words are not required provided the intention is clear. M0A0M1B0 is possible.

Q4. This is a DR question. There are two solutions possible. The first is to rationalise the denominators. To score the M1 we want to see at least one fraction multiplied by the correct fraction $\frac{\sqrt{2}-1}{\sqrt{2}-1}$ *or* $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ *or* $\frac{2-\sqrt{3}}{2-\sqrt{3}}$.
To score the A1 it must be all correct with the denominators seen as 2-1, 3-2, 4-3 or 1,1,1. There must be something there. This leads to the final answer of 1.

The second method is where they try to combine all three fractions. The M1 is given attempting to find a numerator of $(\sqrt{3}+\sqrt{2})(2+\sqrt{3})+(\sqrt{2}+1)(2+\sqrt{3})+(\sqrt{2}+1)(\sqrt{3}+\sqrt{2})$ all multiplied out to $3\sqrt{6}+4\sqrt{3}+5\sqrt{2}+7$ and a denominator of $(\sqrt{2}+1)(\sqrt{3}+\sqrt{2})(2+\sqrt{3})$ multiplied out to $3\sqrt{6}+4\sqrt{3}+3\sqrt{2}+2\sqrt{2}+7$.
The A1 is for getting the correct fraction $\frac{3\sqrt{6}+4\sqrt{3}+5\sqrt{2}+7}{3\sqrt{6}+4\sqrt{3}+5\sqrt{2}+7}$ before cancelling and getting the answer 1.

Q5. This is another DR question. There are 4 marks to get to the integrated expression and 2 marks for finding and checking the value of c.
The first M1 is for du = dx or equivalent (e.g du/dx =1)
The second M1 is for forming the integral. We will allow the absence of du for this mark but sight of dx loses the mark.
The third M1 is for attempting to integrate *their* expression and we must see either a ln u term OR a power of u term.
The A1 is for the correct answer (allow unsimplified terms e.g. ln(c+4+1)) and is dependent on the three M marks. Condone missing brackets for e.g. ln c+4 if recovered.

The next M1 is for setting up an equation to find c using either *their* ln fraction = 3 OR *their* other fraction(s) = $\frac{-1}{3}$ which may be unsimplified).
The final A1 is for correctly finding c =1 AND checking their solution in the other equation or solving the other equation to also get c=1.

Candidates may do it using a different integration method (e.g. integration by parts). Although this is a valid method of integration it is not what the question asks for – 'Using the substitution…' so this is penalized. The first three M marks are lost. Give SC1 for the correct integral answer and the final M1A1 are still available.

Q6. This is another DR question. There are 2 solutions provided.
The first method involves multiplying through by tan x (M1) and the second method involves replacing tan x with sin x/cos x and cot x with cos x/sin x (M1). Both methods then involve getting an equation with 3 terms on one side and zero on the other (M1) and solving their 3-term equation AND get both values of tan x (M1). A correctly solved equation implies any missing M marks. They **must** solve the equation by a valid method (factorisation, formula or completing the square) to get the M1 here.

The A marks are for giving the angles – A1 for any two correct (subject to at least M2) and A2 for all the roots with no additional solutions (subject to M3).

It might be quite common to give M1 M1 M0 A1 A0 because they fail to solve the equation and just write down answers. Note we are allowing 72°/ 252° and also over specified answers. e.g. 71.565°/ 251.565°.

Q7. The first M1 can be awarded for *using* $\sin 8\theta = 2\sin 4\theta\cos 4\theta$ or for *using* $\cos 8\theta = 1 - 2\sin^2 4\theta$ (or equivalent form) or for *using* $\tan 4\theta = \sin 4\theta / \cos 4\theta$. The mark is not available for just writing the formula down.
The second M1 is for using another one.
The final A1 is for using the third one and completing the solution to show it equals 1. (We saw an incorrect factorisation of $\sin 8\theta = 4\sin 2\theta = 4 \times 2\sin\theta\cos\theta$ which qualifies for SC1. A similar approach can be used for $\cos 8\theta$ but the SC1 can only be awarded once).

Some candidates are using small angle approximations which does lead to an answer of 1, but is the incorrect method, so scores M0M0A0.

Q8a. We are expecting to see the 2 equations R cos α = √3 and R sin α = 1 for the first M1. Must be in terms of α.
The B1 mark is for finding R=2. In addition to the method in the MS, they might find R first and use it to solve, say 2 sin α = 1, leading to α = π/6 for the M1A1. The A1 is dependent on scoring M2.

Award M0 if the R is missing. They can still get next M1 but not the A1 (as an M0 has already been given – see section 6c of the marking instructions). M0M1A0B1 is likely to be quite common.

They can get M1 A0 if they draw a right-angled triangle and label it incorrectly giving tan α = √3

Q8b. For the first M1 they use *their* result from part (a) to get an equation equal to √3. An incorrect value of α from Q8a means they can also access the next two M1s as per the MS.
The second M1 is for getting *their* trig expression = a value (probably √3/2).
The third M1 is for solving *their* equation to get at least one solution.
The A1 is given for the two correct values in radians and no others.

Note that candidates who square both sides and solve from there will get extraneous solutions so any of those must be discarded. There is a maximum of M3A0 if they do discard erroneous solutions but only M2A0 if they do not.

Q9a. The first B mark is for finding either f(1) or f(2). The second B mark is for finding the other value AND saying there is a sign change.

Q9b. Any value between 1.125 and 1.25 will do for this mark. Some candidates are just halving the range and using 1.0625 which scores B0.

Q9c. They must give a value for, say f(1.15) = -0.09,(or f(1.15625) = -0.0496, f(1.16) = -0.025, f(1.163) = -0.05, f(1.164) = 0.0015), and a final value of 1.2. The question asks for 1dp so only accept 1.2. 1.2 with no other working scores B0. Other values for f are possible.

Q9di. A change is sign is all that is required here.

Q9dii. We are allowing various comments. See the MS guidance. ISW after a correct answer has been seen.

Q10. The 3 marks are independent.
The first one is for showing the crossing points are 0 and near to +2 and -2.
The second is for rotational symmetry (or close to).
The third is for the general shape of a positive cubic But do not condone incorrect curvature. Some candidates are drawing y=x, which scores B0B1B0.
In all cases use your judgement and mark the intention of the candidate.
List all 3 marks in the order of the MS.

Q11a. The B1 is for differentiating and can be given for seeing x cos x or the unsimplified version sin x + x cos x – sin x.
The M1 is given for x cos x =1 and the final A1 for finishing it off convincingly. If there is no differentiation, then the M1 is not available.

Q11b. There are two methods given.
The first M mark is for attempting to differentiate either 1/x – cos x or x cos x -1.
The first A mark is for setting up the iteration function and it **must** include the subscripts.
The next M mark is for choosing a suitable starting value (accept values between 3.6 and 6.1). They might use x_0 or x_1 or just say 'starting value' – but it will be their first value.
The final A mark is for getting the root 4.9172. The question asked for 4dps. They do not need to show all the working (BC means 'by calculator') to get the A1. If they choose a value outside the range and it gives 4.9172 then give M1 A1. A value outside the range giving the wrong answer will get M0 A0.

Q11c. The comment needs to be more than just 'it is close to another root'.

Q12a. This is a DR question and runs over to page 11 so please check and mark as BP or SEEN if empty and mark if there is something there.
The first M1 is for differentiating y wrt θ. It might be seen on it's own as dy/dθ or as the numerator of dy/dx. We are ignoring dx/dθ unless they try to use it incorrectly in their subsequent work.
The second M1 is for putting their expression equal to 0 AND using either sin 2θ = 2 sin θ cos θ or cos 2θ = one of the correct forms. But if dx /dθ is used incorrectly here then give M0.
The third M1 is for getting terms on one side and factorised. However, some divide through by sin at this point – they can still get the M1 and also the fourth M1. The 3rd and 4th M1s can be implied (BOD) by seeing 2 correct solutions sin θ = 0 and cos θ = - ½.
The 4th M1 is dependent on the previous M1. (We have seen some candidates obtain sin θ = 0 (and possibly cos θ = - ½) from completely wrong working but this does not score).

The A marks are dependent on getting M4 and are for finding exact values of θ and the coordinates. Note that the A marks are all www marks (without wrong working), so dividing through by sin θ will cost all the A marks. Allow equivalent correct forms (e.g. $\frac{4-\sqrt{3}}{2}$ $for\ 2-\frac{\sqrt{3}}{2}$, $\frac{4+\sqrt{3}}{2}$ $for\ 2+\frac{\sqrt{3}}{2}$) for the x-values.

CHECK PAGE 11 for working

Q12b. x=2 is the only acceptable answer for B1.

Section B
Q13. Candidates must use t_1 and t_2 with values 1 and -3 to show t_1t_2 = -3 for the B1. Using -1 and 3 is not correct and scores B0.

Q14. Candidates must use t_1 and t_2 with values 1 and -3 and show the substitution to get to 7.5 for the B1. Using -1 and 3 is not correct and scores B0.

Q15a. The first M1 is for differentiating. We are allowing the gradient as 2at + b as well as 2ax + b. Some are just writing 2ax+b/2at+b and that isn't convincing that they have differentiated – rather just copying from the given answer so give M0.
The second M1 is for substituting into the equation of a straight line using their gradient and x = t. They could use $(y - y_1) = m(x - x_1)$ or y= mx + c. Condone a lack of brackets around their m if recovered.
The final A1 is for getting to the correct answer. As the answer is given, we need to see some working here.

Q15b. The first M1 is for equating their two tangents with distinct values of t. These could be x_p and x_q as per the scheme, but they might choose other pairs of symbols – e.g. P and Q, m and n, t_1 and t_2.
The second M1 is for getting the x terms on one side and the non-x terms on the other.
To get the A1 they must show the working out including the difference of two squares and cancelling to get the required result. Allow the final result in terms of their variable.

Q16. To get the M1 they must work with the two terms with factor a. They may be moved next to each other, but they could also work with them in their original positions.
The A1 is for manipulating the factor a terms correctly. This may be squaring of brackets or difference of two squares. But it must be convincing to get the required result. Give A0 for any error seen in the working.

Q17. The first M1 is for differentiating to get either 2x or 2t.
The second M1 is for using the negative reciprocal to get -1/their gradient.
The A1 is for substituting into $(y - y_1) = m(x - x_1)$ or y= mx + c to get the required result or equivalent unsimplified. It must be convincing as the answer is given. Any error seen is A0.

Q18. They must use the formula from line 19 of the article. The M1 is for substituting a=6 and h=5 and the A1 is for the correct answer. Allow 2 marks for a completely correct valid alternative method.

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